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Meson spectroscopy from holomorphic probes on the warped deformed conifold

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Abstract: We study D7 brane probes holomorphically embedded in the Klebanov-Strassler model. Analyzing the κ-symmetry condition for D7 branes wrapping a 4-cycle of a deformed conifold we find configurations that do not break $\mathcal{N}=1$ supersymmetry of the background. We compute the fluctuations of the probe around one of these configurations and obtain the spectrum of vector and scalar flavored mesons in the dual gauge theory. The spectrum is discrete and exhibits a mass gap.

Keywords: D-branes, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

The AdS/CFT correspondence [1]–[3] (see [4] for a review) is a conjectured equivalence between a string theory (type IIB on $AdS_5 \times S^5$) and a gauge theory ($\mathcal{N} = 4$ SU(N_c) SYM in four dimensions). In the the large 't Hooft coupling limit ($g^2_{\rm YM} N \to \infty$) we can neglect the string massive modes using type IIB supergravity on the string side of the correspondence.

Since the formulation of the AdS/CFT correspondence its extension to theories with less supersymmetries and with no conformal invariance has been of great interest. In particular, there are well known examples of supergravity duals of $\mathcal{N} = 1$ gauge theories [5, 6] (see [7]–[9] for a review). Unfortunately, most of the known supergravity backgrounds do not include quarks in the fundamental representation. In the large N limit the Feynman diagrams of Yang-Mills theory are related to a genus expansion of closed strings. Adding fundamental flavors in the gauge theory introduces boundaries in this expansion. It means that on the string side we have to incorporate Dp brane. If the number of these branes N_f is much smaller than N_c we can neglect the back-reaction of the brane probe on the supergravity bulk geometry.

JHEP03(2005)014

Adding fundamental flavors in the gauge theory requires adding of $4d$ space-time filling Dp branes in the bulk. In order to avoid a problem with tadpoles the net charge of a space-time filling brane has to cancel. On the other hand, the RR charge is necessary for the brane stability. It was proposed in [10] (see also [11]) that one can solve this puzzle by having a supersymmetric Dp brane wrapping a topologically trivial cycle in the internal space. The configuration is stable provided that the masses of the modes controlling the slipping of the probe off the cycle are above the BF bound. This idea was further explored by [12] for a D7 probe on $AdS_5 \times S^5$. The spectrum of mesons with arbitrary R-charge in $\mathcal{N} = 2$ SYM was extracted from the masses of the probe fluctuations. It was shown that the spectrum is discrete with a mass gap. For related works, see [13]–[21].

The main purpose of this paper is to explore supersymmetric space-time filling D7 brane probes in the Klebanov-Strassler (KS) model [6]. In this supergravity background the metric has a standard D3-form and the internal part of the metric is given by the $6d$ deformed conifold defined by:

$$\sum_{i=1}^{4} z_i^2 = \epsilon^2 \,. \tag{1.1}$$

The background involves also fractional D5-branes wrapped around a shrinking S^2 of the conifold (see [22] for the correspondence between various gauge theory objects and wrapped Dp brane probes in this model). It was argued in [10] that making the quark mass very large should decouple the quarks from the IR theory. Therefore the D7 brane probe should be space-time filling in UV, while ending at a finite distance at the radial direction to be absent in the IR. This distance corresponds to the mass of the quarks in the gauge theory and should appear as a free parameter in a solution of the probe equations of motion. It was suggested in [10] that in the KS model the following configuration yields a supersymmetric D7 embedding:

$$z_3^2 + z_4^2 = \lambda^2 \,. \tag{1.2}$$

Here λ^2 is a constant parameter. Under T-duality this is mapped into type IIA D6 branes [23]. In this paper we will demonstrate that this embedding indeed satisfies the κ-symmetry condition and therefore solves the equations of motion of the space-filling D7 probe.

A different D7 probe configuration was studied in [24]. In terms of the coordinates z_i's it may be written as:

$$\mathrm{Im}\left(z_1^2 + z_2^2\right) = 0 \,. \tag{1.3}$$

It was explicitly shown that this embedding solves the probe equations of motion. Furthermore, computing the quadratic fluctuations of the probe the authors of [24] found the masses of vector and pseudo-scalar mesons in the dual confining gauge theory. Although the embedding considered in this paper is not holomorphic and therefore breaks supersymmetry, the solution is stable and the meson spectra exhibit a mass gap. The solution has no free mass parameter and the brane extends to the IR region.

For $\epsilon = 0$ the equation (1.1) defines a cone over $T_{1,1}$ (the conifold) and the KS solution reduces to the so called Klebanov-Tseytlin (KT) solution [25], which has a singularity at the tip of the conifold. The holomorphically embedded D7 in this background was considered in [26]. In terms of the coordinates used in (1.1) the probe discussed in this paper is given by:

$$z_3 + iz_4 = m^2\,, \tag{1.4}$$

where m was identified in [26] as the quark mass. The supersymmetric supergravity solution for the leading back-reaction effects was also found. This approximation, however, is valid only in the UV region.

Without fractional branes the KT solution reduces further to the Klebanov-Witten solution. In this model the stack of D3 branes warps the conifold metric and the resulting geometry is just $AdS_5 \times T_{1,1}$. Following the ideas of [27, 28] it was argued in [23] that D3 branes wrapping 3-cycles of $T_{1,1}$ are dual to dibaryon operators in the gauge theory (see also [29]–[31]). Furthermore, a D5 brane wrapping 2-cycles of $T_{1,1}$ behaves as a domain wall in AdS_5. The idea to use D7 branes to add fundamental quarks to the background was proposed by [10] (see [32] for the recent progress).

Recently the authors of [33] found a rich family of supersymmetrically embedded D5-branes in the Maldacena-Nunez (MN) background [5]. This was achieved by solving equations arising from the κ-symmetry condition. The solutions involve a free parameter related to the mass of the fundamental quarks. Moreover, exploring the quadratic fluctuations around these brane configurations admits a quite simple relation between the meson masses and the mass of the fundamentals.

In this paper will study space-filling supersymmetric D7 probes embedded in the KS solution. Similarly to [33] our main technique is κ-symmetry. Throughout this paper we will mostly study the embedding defined by:

$$z_4 = \mu\,, \tag{1.5}$$

but we will also argue that the probe configurations (1.2) and (1.4) do not spoil supersymmetry. More generally we will show that the κ-symmetry condition is fulfilled provided that the embedding can be written in the form $z_4 = f(z_1^2 + z_2^2)$, where φ is an arbitrary holomorphic function. This trivially holds for (1.2), (1.4) and (1.5). Remarkably, the embedding (1.5) satisfies the condition of [10]. Indeed, we will demonstrate that for $\mu > \epsilon$ the D7 probe ends at a finite distance from the IR. We preferred to explore the embedding (1.5), since this is the only configuration that leaves an SO(3) $\subset$ SO(4) unbroken, so that the coordinate choice becomes relatively simple. Using this coordinates we were able to find the spectra of the vector and scalar mesons for the special case $\mu = 0$. The spectra exhibit a mass gap and the mass scale is that of the glueballs.

The organization of the paper is as follows. In section 2 we give a short review of the deformed conifold and the KS model. We find it useful to re-write the metric and the forms of the model in terms of the SU(2) invariant 1-forms used in [34]. Based on these 1-forms we find a convenient coordinate parameterization, which properly describes the embedding (1.5). We also comment on the geometrical structure of the probe configuration

JHEP03(2005)014

for different μ's. In section 3 we use the SO(4) invariant representation of the B-field and the metric to argue that the configurations (1.2), (1.4) and (1.5) satisfy the κ-symmetry condition originally derived in [35]. In section 4 we present the D7 probe action in the case of the embedding (1.5) with $\mu = 0$. As usual the action includes the DBI and WZ parts. We then compute the vector and the scalar meson spectra by solving the equations for the brane quadratic fluctuation. The set of fields appearing in the action include the scalar field controlling the brane fluctuation around the surface $z_4 = 0$, gauge fields directed along the 4-cycle wrapped by the brane and gauge fields directed parallel to the $4d$ boundary of the $10d$ metric. We will see that there is a mixing between the scalar and one of the gauge fields in the equations of motion. In section 5 we summarize our results and discuss various open questions. We summarize some useful formulae in the appendix A and B.

2. The geometry of a deformed conifold and the Klebanov-Strassler model

In this section we will describe the deformed conifold geometry and the KS background using an alternative formulation, which is advantageous for the description of the embedding $z_4 = \mu$. We will also comment on the related deformation in the dual gauge theory.

2.1 The deformed conifold and the embedding $z_4 = \mu$

The deformed conifold is defined by (1.1). It can conveniently be re-written as:

$$\det W = -\epsilon^2 \,, \qquad \text{where} \qquad W = \begin{pmatrix} z_3 + iz_4 & z_1 - iz_2 \\ z_1 + iz_2 & -z_3 + iz_4 \end{pmatrix} . \tag{2.1}$$

In term of this matrix the radial coordinate r along the conifold is given by:

$$r^2 = \frac{1}{2}\mathrm{Tr}\left(W^\dagger W\right) = \sum_{i=1}^{4} |z_i|^2 \,. \tag{2.2}$$

In this paper we will use the standard re-definition of the radial coordinate according to $r^2 = \epsilon^2 \cosh\tau$. The minimal value of the radial coordinate is $r_{\mathrm{min}} = \epsilon$ (or $\tau = 0$). It corresponds to the "tip", where the S^2 shrinks to zero. If we identify $\mathcal{F} = \mathcal{F}(r^2)$ with the Kähler potential, then the metric on the deformed conifold is [36]:

$$\mathrm{d}s_6^2 = \partial_\alpha \bar{\partial}_\beta \mathcal{F} \mathrm{d}z_\alpha \mathrm{d}\bar{z}_\beta = \frac{1}{4}\mathcal{F}'' \left|\mathrm{Tr}\left(W^\dagger \mathrm{d}W\right)\right|^2 + \frac{1}{2}\mathcal{F}'\mathrm{Tr}\left(\mathrm{d}W^\dagger \mathrm{d}W\right) . \tag{2.3}$$

The following parameterization is used to express the metric in terms of the radial and five angular coordinates:

$$W = L_1 W^{(0)} L_2^\dagger \qquad \text{where} \qquad W^{(0)} \equiv \begin{pmatrix} 0 & \epsilon e^{\tau/2} \\ \epsilon e^{-\tau/2} & 0 \end{pmatrix} \tag{2.4}$$

and the SU(2) matrices L_1 and L_2 depend only on the angular coordinates:

$$L_i = \begin{pmatrix} \cos\frac{\theta_i}{2} e^{\frac{i}{2}(\psi_i + \phi_i)} & -\sin\frac{\theta_i}{2} e^{-\frac{i}{2}(\psi_i - \phi_i)} \\ \sin\frac{\theta_i}{2} e^{\frac{i}{2}(\psi_i - \phi_i)} & \cos\frac{\theta_i}{2} e^{-\frac{i}{2}(\psi_i + \phi_i)} \end{pmatrix} \qquad \text{where} \qquad i = 1, 2 \,. \tag{2.5}$$

JHEP03(2005)014

Since the matrix W depends only on the sum $\psi_1 + \psi_2$ one usually sets $\psi_1 = \psi_2 = \frac{1}{2}\psi$. Next we introduce the SU(2) left invariant 1-forms $\{h_i, \tilde{h}_i\}_{i=1,2,3}$ related to the matrices $L_{1,2}$ through:

$$L_1^\dagger \mathrm{d}L_1 = \frac{i}{2} h_i \sigma^i \qquad \text{and} \qquad L_2^\dagger \mathrm{d}L_2 = \frac{i}{2} \tilde{h}_i \sigma^i \,, \tag{2.6}$$

where $\sigma_{i=1,2,3}$ are the Pauli matrices. The explicit formulae for the forms in terms of the angular coordinates appear in appendix A. In appendix B we show how these forms are connected to the forms g_i's used in [6]. These 1-forms satisfy the SU(2) $\times$ SU(2) Maurer-Cartan equations:

$$\mathrm{d}h_i = \frac{1}{2}\epsilon_{ijk} h_j \wedge h_k, \qquad \mathrm{d}\tilde{h}_i = \frac{1}{2}\epsilon_{ijk} \tilde{h}_j \wedge \tilde{h}_k \,. \tag{2.7}$$

Requiring the metric to be Ricci flat leads to a differential equation for the Kähler potential $\mathcal{F}(r^2)$ as described in [36] (see also [37, 38]). In terms of the 1-forms the metric of the deformed conifold takes the following form:

$$\begin{aligned} \epsilon^{-4/3} ds_6^2 = B^2(\tau)\left(\mathrm{d}\tau^2 + \left(h_3 + \tilde{h}_3\right)^2\right) + \\ + A^2(\tau)\left(h_1^2 + h_2^2 + \tilde{h}_1^2 + \tilde{h}_2^2 + \frac{2}{\cosh\tau}\left(h_2\tilde{h}_2 - h_1\tilde{h}_1\right)\right), \end{aligned} \tag{2.8}$$

where

$$A^2(\tau) = \frac{2^{-1/3}}{4}\coth\tau\,(\sinh(2\tau) - 2\tau)^{1/3} \qquad \text{and} \qquad B^2(\tau) = \frac{2^{2/3}}{6}\frac{\sinh^2\tau}{(\sinh(2\tau) - 2\tau)^{2/3}} \,. \tag{2.9}$$

Let us now demonstrate how using the 1-forms $\{h_i, \tilde{h}_i\}$ one can parameterize the surface $z_4 = \mu$ embedded in the deformed conifold. Let us start from the $\mu = 0$ case. Since $z_4 = \frac{1}{2i}\mathrm{Tr}W$ and $\mathrm{Tr}W^{(0)} = 0$ the job is done by an identification of the SU(2) matrices L_1 and L_2:

$$L_1 = L_2 \,. \tag{2.10}$$

In terms of the angular coordinates it implies that $\theta_1 = \theta_2$, $\phi_1 = \phi_2$, so that $h_i = \tilde{h}_i$ for $i = 1,2,3$. We obtain a $4d$ surface with $z_4 = 0$ along it. For $\mu \neq 0$ we set

$$L_2 = L_1 S \qquad \text{with} \qquad S = \begin{pmatrix} \cos\frac{\gamma}{2}e^{i\frac{\delta}{2}} & -i\sin\frac{\gamma}{2}e^{-i\frac{\delta}{2}} \\ -i\sin\frac{\gamma}{2}e^{i\frac{\delta}{2}} & \cos\frac{\gamma}{2}e^{-i\frac{\delta}{2}} \end{pmatrix}. \tag{2.11}$$

Substituting this into (2.4) we obtain:

$$z_4 = \frac{1}{2i}\mathrm{Tr}W = \epsilon \sin\frac{\gamma}{2}\cosh\left(\frac{\tau}{2} + i\frac{\delta}{2}\right). \tag{2.12}$$

It means that for real μ the embedding $z_4 = \mu$ corresponds to:

$$\sin\left(\frac{\gamma(\tau)}{2}\right) = \frac{\mu}{\epsilon}\frac{1}{\cosh\frac{\tau}{2}} \qquad \text{and} \qquad \delta = 0 \,. \tag{2.13}$$

We see that for $\mu > \epsilon$ the minimal value of the radial coordinate τ along the surface $z_4 = \mu$ is $\tau_{\mathrm{min}} = 2\cosh^{-1}(\frac{\mu}{\epsilon})$ and the D7 brane probe embedded in this way does not reach the "tip" located at $\tau = 0$. On the other hand, for $\mu < \epsilon$ we have $\tau_{\mathrm{min}} = 0$. This is shown

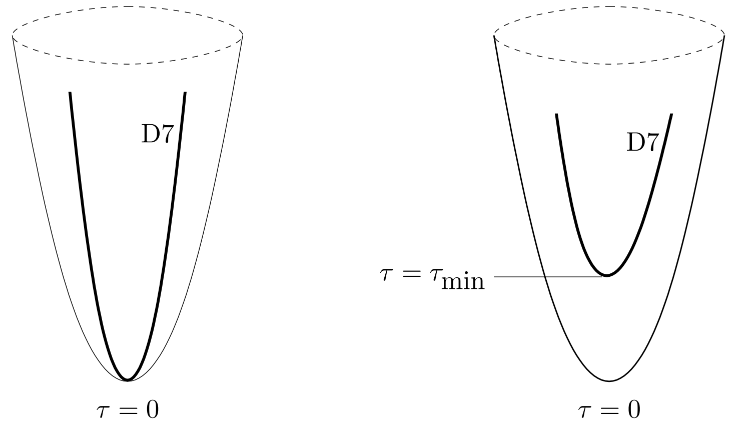

Figure 1: The D7 probe configuration $z_4 = \mu$ on a deformed conifold for $\mu < \epsilon$ (left) and for $\mu > \epsilon$ (right).

schematically on figure 1. One can arrive at the same result directly from the definition of the deformed conifold (1.1). Indeed, along the surface $z_4 = \mu$ the radial coordinate r satisfies:

$$r^2 = \sum_{i=1}^{3} |z_i|^2 + \mu^2 \geq \left| \sum_{i=1}^{3} z_i \right|^2 + \mu^2 = \left|\epsilon^2 - \mu^2\right| + \mu^2 = \begin{cases} \epsilon^2 & \text{for} \quad \mu \leq \epsilon \\ 2\mu^2 - \epsilon^2 & \text{for} \quad \mu \geq \epsilon \end{cases} . \tag{2.14}$$

Recalling that $r^2 = \epsilon \cosh \tau$ we see this is identical to (2.13). It turns out that the same conclusion holds for the configurations (1.2) and (1.4). For instance, with μ replaced by λ the relation (2.14) holds for the case (1.2). Notice that this result matches the prediction of (1.2). Indeed, we see that the D7 brane configuration has a free parameter μ and making this parameter large enough ($\mu > \epsilon$) we find that the probe brane ends at a finite distance at the radial coordinate τ ($\tau_{\min} > 0$) and so is absent in the IR.

Using the relation (2.11) we can express the forms $\tilde{h}_1$, $\tilde{h}_2$ and $\tilde{h}_3$ in terms of the forms h_1, h_2 and h_3 and the coordinates γ and δ. Substituting (2.11) into the second equation in (2.6) we get:

$$\begin{aligned} \tilde{h}_1 &= (h_1 - \mathrm{d}\gamma)\cos\delta - (h_3 \sin\gamma + h_2\cos\gamma)\sin\delta \\ \tilde{h}_2 &= (h_1 - \mathrm{d}\gamma)\sin\delta + (h_3 \sin\gamma + h_2\cos\gamma)\cos\delta \\ \tilde{h}_3 &= (h_3\cos\gamma - h_2\sin\gamma) + \mathrm{d}\delta \,. \end{aligned} \tag{2.15}$$

As a consistency check one may verify that the forms $\tilde{h}_i$'s satisfy the Maurer-Cartan equations (2.7).

The D7 brane in our setup spans the world-volume of the D3 branes and wraps the 4-cycle defined by $z_4 = \mu$. For this configuration the coordinates γ and δ satisfy (2.13). We therefore will refer to the coordinates x_μ, τ and $h_{i=1,2,3}$ as the world-volume coordinates of the branes, while the coordinates γ and δ will be regarded as the transverse coordinates. Note again that the coordinate γ is constant along the brane only for $\mu = 0$.

JHEP03(2005)014

Let us end this section with a remark on the geometry of the embedding $z_4 = \mu$. For $\mu \neq 0$ the configuration admits the SU(2) isometry, which is the subgroup of the isometry group SO(4) = SU(2) $\times$ SU(2) of the deformed conifold. This is similar to the isometry subgroup SU(2) $\subset$ SO(4) preserved by the D7 probe considered in [24]. It is therefore attempting to compare these two configurations. For $\epsilon \to 0$ the type IIA dual picture of the singular locus of the conifold takes the form of a pair of perpendicular NS5 branes [39]–[41]. For the deformed conifold ($\epsilon \neq 0$) there is a "diamond" structure at the intersection of the NS5 branes [42]. Under T-duality the fractional D3 branes of the KS background are transformed to D4 branes connecting the NS5 branes. Adding D6 branes on the top of the NS5 branes one introduces into the setup flavored fundamental quarks, which correspond to the strings between the D6 and the D4 branes [43]. In the T-dual picture it gives D7 branes that intersect the singular locus. The transverse coordinates of the D7 brane are the S^2 coordinates of the base. Using the deformed conifold formulation of [44] this structure was explicitly realized in [24]. It was further shown that the configuration has a form of two D7 branes given by two cylinders smoothly intersecting each other at a circle at $\tau = 0$. This circle is embedded in the S^3 and corresponds to the "diamond". Apart from the SU(2) isometry preserved by the configuration there is a residual U(1) symmetry one gets after placing the pair of the D7 branes at the poles of the S^2. This U(1) is not a symmetry of the full background, but rather of the induced metric on the D7 brane probe. For the embedding $z_4 = \mu$ it is parallel to the U(1) isometry of the induced metric we obtain in the special case $\mu = 0$. In terms of the angular coordinates introduced above this isometry corresponds to the invariance under $\delta \to \delta + \mathrm{const}$. In our setup, however, we cannot associate directly the transverse coordinates (γ and δ) introduced in this section to the coordinates of the S^2 and the 1-forms h_i's to the the forms of the S^3. To see this, let us consider pullback of the metric (2.8) in the case of the embedding $z_4 = \mu$ for $\mu = 0$. According to (2.15) this embedding corresponds to $h_i = \tilde{h}_i$ and near $\tau = 0$ we obtain:

$$ds_6^2 = \frac{\epsilon^{4/3}}{2^{5/3}3^{1/3}} \left(\mathrm{d}\tau^2 + 4(h_3^2 + h_2^2) \right) + O(\tau^2)\,. \tag{2.16}$$

We see that it does not reproduce the metric of the non-shrinking S^3. We therefore cannot use directly the "diamond" structure to argue that our D7 brane probe gets mapped into the type IIA D6 brane. On the other hand, as we will discuss in the next section the holomorphic structure of the embedding is necessary to preserve the supersymmetry of the KS solution. The only holomorphic embedding with the SU(2) isometry is $z_i = \mathrm{const}$. It will be very interesting to understand how this holomorphic embedding posses the T-dual description using the "diamond" structure of the conifold similarly to [24].

2.2 The $10d$ metric and the forms in the KS background

Now we are in a position to present the constituents of the KS background in terms of the radial coordinate τ, the forms $h_{i=1,2,3}$ and the coordinates γ and δ. The $10d$ metric and the 5-form flux in the KS solution have the structure of the D3 brane solution, namely:

$$\mathrm{d}s^2 = h^{-1/2} \left(\mathrm{d}x_0^2 + \cdots + \mathrm{d}x_3^2 \right) + h^{1/2} \mathrm{d}s_{M_6}^2 \tag{2.17}$$

and

$$\tilde{F}_5 = \frac{1}{g_s}(1+\star_{10})\mathrm{d}h^{-1}\wedge \mathrm{d}x_0\wedge\cdots\wedge \mathrm{d}x_3\,, \tag{2.18}$$

where M_6 is the deformed conifold metric (2.8) and the harmonic function h depends only on the coordinate τ:

$$h(\tau) = \frac{\epsilon^{-4/3}}{m_{\text{gb}}^2} I(\tau), \qquad \text{where} \quad I(\tau) = \int_\tau^\infty dx \frac{x\coth x - 1}{\sinh^2 x}(\sinh(2x)-2x)^{1/3}\,. \tag{2.19}$$

Here m is the scale mass of glueballs (see [45] and [46]) given by

$$m_{\text{gb}}^2 = \frac{\epsilon^{4/3}}{2^{2/3}(g_s M\alpha')^2}\,, \tag{2.20}$$

where M is the number of the fractional D3 branes wrapping the shrinking S^2. The five form $\tilde{F}_5$ in (2.18) related to the RR 4-form C_4 by

$$\tilde{F}_5 = F_5 + B_2\wedge \mathrm{d}C_2\,, \qquad \text{where} \qquad F_5 = \mathrm{d}C_4\,. \tag{2.21}$$

Here C_2 is the RR 2-form. In our case the RR 4-form is defined by:

$$C_4 = \frac{1}{g_s}h^{-1}\mathrm{d}x_0\wedge\cdots\wedge \mathrm{d}x_3\,. \tag{2.22}$$

Next we consider the NS B-field. In terms of the 1-forms h_i's and the coordinates γ and δ it reads:

$$\begin{aligned} B_2 = \frac{1}{4}M\alpha'(f-k)\Bigg[\cosh\tau\Bigg(&(1-\cos(\gamma))\,h_1\wedge h_2 - \sin(\gamma)h_3\wedge \mathrm{d}\gamma - \\ &- \sin(\gamma)h_1\wedge h_3 - \cos(\gamma)h_2\wedge \mathrm{d}\gamma\Bigg) + \\ &+\Bigg((\sin(\delta)h_1+\cos(\delta)h_2)\wedge(h_1-\mathrm{d}\gamma) + \\ &+(\cos(\delta)h_1-\sin(\delta)h_2)\wedge(\sin(\gamma)h_3+\cos(\gamma)h_2)\Bigg)\Bigg]\,, \end{aligned} \tag{2.23}$$

where

$$f(\tau) = \frac{\tau\coth\tau - 1}{2\sinh\tau}(\cosh\tau - 1)\,, \qquad k(\tau) = \frac{\tau\coth\tau-1}{2\sinh\tau}(\cosh\tau+1)\,. \tag{2.24}$$

We learn from (2.23) that for $\mu=0$ (or equivalently for $\gamma,\delta=0$) the B-field vanishes along the brane probe. Similarly to (2.23) one can re-write C_2 using the coordinates introduced in this section. The final expression is, however, even more complicated than (2.23) and we will not give it here since (as we will argue in the next section) the RR form C_2 does not contribute to the WZ action and therefore is irrelevant for the present discussion.

We will later need the expression for the RR 6-form C_6. The corresponding 7-form field strength is defined by:

$$F_7 = \star_{10}F_3 - C_4\wedge H_3\,. \tag{2.25}$$

JHEP03(2005)014

The equation of motion of C_2 implies that $\mathrm{d}F_7 = 0$. Here $\star_{10}$ denotes the $10d$ Hodge dual. Denoting by $\star_6$ the Hodge dual on M_6 we may re-write the first term in (2.25) as:

$$\star_{10} F_3 = h^{-1} \star_6 F_3 \wedge \mathrm{d}x_0 \wedge \cdots \wedge \mathrm{d}x_3 \,. \tag{2.26}$$

Remarkably, the 3-forms F_3 and H_3 in the KS background satisfy the following relations:

$$\star_6 F_3 = g_s^{-1} H_3 \qquad \text{and} \qquad \star_6 H_3 = -g_s F_3 \,. \tag{2.27}$$

Plugging this into the definition (2.25) of F_7 and using the expression (2.22) for C_4 we arrive at the conclusion that $F_7 = 0$ in the KS solution. We thus set $C_6 = 0$ in the rest of this paper.

2.3 The dual gauge theory

The dual field theory in the KS model is a 4d $\mathcal{N} = 1$ SU$(N+M) \times$ SU(N) gauge theory with a SU(2) $\times$ SU(2) global symmetry inherited from the conifold isometries. Here N and M are the numbers of the physical and the fractional D3 branes respectively. M is fixed by the charge of the RR 3-form, while N is encoded in the UV behavior of the 5-form (2.18). The gauge theory is coupled to two bi-fundamental chiral multiplets $A_{i=1,2}$ and $B_{i=1,2}$, which transform as a doublet of one of the SU(2)'s each and are inert under the second SU(2). The superpotential inert under the global symmetries is:

$$W_{\text{conifold}} \sim \epsilon^{ij}\epsilon^{kl} \mathrm{Tr}\,(A_i B_j A_k B_l) \,. \tag{2.28}$$

This theory is believed to exhibit a cascade of Seiberg dualities reducing in the deep IR to pure SU(M). There is a simple identification of the superfields $A_{i=1,2}$ and $B_{i=1,2}$ in terms of the coordinates z_i's:

$$w_1 = A_1 B_1 \,, \qquad w_2 = A_2 B_2 \,, \qquad w_3 = A_1 B_2 \,, \qquad w_4 = A_2 B_1 \,,$$

where

$$w_{1,2} = \pm z_3 + i z_4 \qquad \text{and} \qquad w_{3,4} = z_1 \pm i z_2 \,. \tag{2.29}$$

Since $z_4 \sim w_1 + w_2$ the natural deformation of the superpotential (2.28) corresponding to the embedding $z_4 = \mu$ is [26, 20]:

$$W = W_{\text{conifold}} + \lambda Q \,(A_1 B_1 + A_2 B_2 - \mu)\, \tilde{Q} \,. \tag{2.30}$$

Recall that a position of a probe D3 brane on the conifold is encoded in the vacuum expectation values of the fields $A_{1,2}$ and $B_{1,2}$. We see that the superpotential implies that the fundamentals (arising as $3-7$ strings) become massless, when the probe D3 brane is on the D7 brane locus and therefore $z_4 - \mu = 0$ as expected. For the motivation of this deformation using the theory of D3 on $\mathcal{C}^2/\mathcal{Z}_2$ see [26].

3. κ-symmetry

As mentioned in the Introduction to get a supersymmetric brane embedding the Killing spinor ε of the KS solution has to satisfy the κ-symmetry condition [47]–[50]:

$$\Gamma_\kappa \varepsilon = \varepsilon\,, \tag{3.1}$$

where the matrix Γ_κ depends on the geometry of the embedding as well as on the world-volume gauge fields of the brane. It can be expressed in the following way:

$$\Gamma_\kappa = e^{-a/2}\Gamma'_{(0)}e^{a/2}\,, \tag{3.2}$$

where a contains all the dependence on the B-field and the world-volume gauge field. More precisely it depends on the modified 2-form field strength $\mathcal{F} = \varphi^\star(B) + 2\pi\alpha' F$, where $\varphi^\star(B)$ is the pullback of the B-field. The matrix $\Gamma'_{(0)}$ depends on the Dp brane embedding and for $p = 7$ it is given by:

$$\Gamma'_{(0)} = i\sigma_2 \otimes \frac{1}{8!\sqrt{|g|}}\epsilon^{i_1\dots i_8}\partial_{i_1}X^{n_1}\cdot\dots\cdot\partial_{i_8}X^{n_8}\gamma_{n_1\dots n_8} \qquad \text{with} \qquad \gamma_n = E^a_n\Gamma_a\,. \tag{3.3}$$

Here γ_n and Γ_a's are the $10d$ curved and flat space gamma-matrices respectively and $|g|$ denotes the determinant of the induced metric.

The $10d$ supergravity spinor of the KS solution can be decomposed in the following way:

$$\varepsilon = \zeta \otimes \chi\,, \tag{3.4}$$

where ζ is a four dimensional chiral spinor ($\Gamma^4\zeta = \zeta$) and χ is a six dimensional chiral spinor ($\Gamma^6\chi = -\chi$). It was shown in [51] that for a type IIB background of the form (2.17), (2.17) the spinor χ is given by the Killing spinor $\tilde{\chi}$ of the $6d$ internal space (the deformed conifold in the KS background) multiplied by a power of the warp function $h(\tau)$:

$$\tilde{\chi} = h^{-1/8}\chi\,. \tag{3.5}$$

This result continues to hold also in the presence of the 3-form $G_3 \equiv F_3 + \frac{i}{g_s}H_3$ as in the KS solution. Moreover the supersymmetry associated with χ is unbroken provided that this 3-form is of type $(2,1)$ and is imaginary self dual ($\star_6 G_3 = iG_3$) [52, 53].

On substituting the spinor (3.4) into the the κ-symmetry condition (3.1) and considering the case where the D7 brane spans the D3 world-volume coordinates in (2.17) and wraps a 4-cycle of the internal space, we see that the four dimensional part of the condition corresponding to the spinor ζ is trivially satisfied and (3.1) effectively reduces to the condition on a D3 brane wrapping the 4-cycle. This problem was discussed in [35], where it was proven that in order to preserve the supersymmetry related to $\tilde{\chi}$ the solution has to satisfy the following three restrictions:

1. The embedding is holomorphic [54].

2. The modified 2-form field strength $\mathcal{F} = \varphi^\star(B) + 2\pi l_s^2 F$ is of type $(1,1)$. Here $\varphi^\star(B)$ is the pullback of the 2-form B.

3. The pullback of the Kähler 2-form J and the form $\mathcal{F}$ satisfy the equation:

$$\varphi^\star(J) \wedge \mathcal{F} = \tan\theta \left(\mathrm{vol}_4 - \frac{1}{2}\mathcal{F} \wedge \mathcal{F} \right) , \tag{3.6}$$

where $\mathrm{vol}_4 = \frac{1}{2}\varphi^\star(J) \wedge \varphi^\star(J)$ is the canonical volume element of the 4-cycle and θ is a constant parameter.

In the rest of this section we will examine these conditions for the deformed conifold case. To this end we will need the expression for the B-field in terms of the complex coordinates z_i's [55]:

$$B = b(\tau)\epsilon_{ijkl} z_i \bar{z}_j \mathrm{d}z_k \wedge \mathrm{d}\bar{z}_l \qquad \text{with} \qquad b(\tau) = \frac{i g_s M \alpha'}{2\epsilon^4} \frac{\tau \coth\tau - 1}{\sinh^2\tau} . \tag{3.7}$$

This expression is explicitly SO(4) invariant. Moreover, in this gauge the B-field is of type $(1,1)$ and so for vanishing $F_{\mu\nu}$ and for an arbitrary holomorphic embedding the 2-form $\mathcal{F}$ also of type $(1,1)$ in agreement with the supersymmetry restriction described above.

Now let us consider the condition (3.6). We want to show that the embedding $z_4 = \mu$ is a solution of (3.6) for $\theta = 0$. We will assume also that $F_{\mu\nu} = 0$. Plugging $z_4 = \mu$ into (3.7) and expressing z_3 in terms of z_1 and z_2 we get:

$$\begin{aligned} \varphi^\star(B) = \mu^2 b(\tau) \cdot \Bigg[&-(z_2 - \bar{z}_2)\left(\frac{z_1}{z_3} - \frac{\bar{z}_1}{\bar{z}_3}\right) \mathrm{d}z_1 \wedge \mathrm{d}\bar{z}_1 + (z_1 - \bar{z}_1)\left(\frac{z_2}{z_3} - \frac{\bar{z}_2}{\bar{z}_3}\right) \mathrm{d}z_2 \wedge \mathrm{d}\bar{z}_2 + \\ &+ \left(z_3 - \bar{z}_3 + (z_1 - \bar{z}_1)\frac{z_1}{z_3} + (z_2 - \bar{z}_2)\frac{\bar{z}_2}{\bar{z}_3} \right) \mathrm{d}z_1 \wedge \mathrm{d}\bar{z}_2 - \\ &- \left(z_3 - \bar{z}_3 + (z_1 - \bar{z}_1)\frac{\bar{z}_1}{\bar{z}_3} + (z_2 - \bar{z}_2)\frac{z_2}{z_3} \right) \mathrm{d}z_2 \wedge \mathrm{d}\bar{z}_1 \Bigg] . \end{aligned} \tag{3.8}$$

Similarly the pullback of the Kähler 2-form is:

$$\begin{aligned} \varphi^\star(J) &= \partial_\alpha \bar{\partial}_\beta \mathcal{F} \mathrm{d}z_\alpha \wedge \mathrm{d}\bar{z}_\beta \big|_{z_4=\mu} \\ &= \mathcal{F}'' \left(2\,|z_1|^2\, \mathrm{d}z_1 \wedge \mathrm{d}\bar{z}_1 + 2\,|z_2|^2\, \mathrm{d}z_2 \wedge \mathrm{d}\bar{z}_2 + z_1\bar{z}_2 \mathrm{d}z_1 \wedge \mathrm{d}\bar{z}_2 + z_2\bar{z}_1 \mathrm{d}z_2 \wedge \mathrm{d}\bar{z}_1 \right) + \\ &\quad + \mathcal{F}' \Bigg(\left(1 + \frac{|z_1|^2}{|z_3|^2}\right) \mathrm{d}z_1 \wedge \mathrm{d}\bar{z}_1 + \left(1 + \frac{|z_2|^2}{|z_3|^2}\right) \mathrm{d}z_2 \wedge \mathrm{d}\bar{z}_2 + \\ &\qquad + \frac{z_1\bar{z}_2}{|z_3|^2} \mathrm{d}z_1 \wedge \mathrm{d}\bar{z}_2 + \frac{z_2\bar{z}_1}{|z_3|^2} \mathrm{d}z_2 \wedge \mathrm{d}\bar{z}_1 \Bigg) . \end{aligned} \tag{3.9}$$

This is now a straightforward exercise to verify that $\varphi^\star(J) \wedge \varphi^\star(B) = 0$ and thus the condition (3.6) is fulfilled for the embedding $z_4 = \mu$. There is, however, a simple symmetry argument one can use to significantly simplify the proof. Both the B-field and the Kähler form are SO(4) invariant objects. In particular, they are invariant under rotations in the (z_1, z_2) plane. On the other hand, the Kähler form is also inert under

$$z_1 \to z_2 \qquad z_2 \to z_1 , \tag{3.10}$$

while the B-field transforms under this reflection as $B \to -B$. Furthermore, we have $\varphi^\star(J) \wedge \varphi^\star(B) = \phi \cdot \mathrm{d}z_1 \wedge \mathrm{d}z_2 \wedge \mathrm{d}\bar{z}_1 \wedge \mathrm{d}z_2$ for some function $\phi = \phi(z_1, z_2, \bar{z}_1, \bar{z}_2)$. Since the embedding $z_4 = \mu$ is both invariant under rotations and the reflection (3.10) we conclude that ϕ is invariant under rotations and transforms as $\phi \to -\phi$ under (3.10). The only functions with these two properties is $\phi \equiv 0$.

Remarkably, this proof is valid for a general holomorphic embedding of the form:

$$z_4 = f\left(z_1^2 + z_2^2\right). \tag{3.11}$$

In particular, the configurations (1.2) and (1.4) are of this form and therefore satisfy the κ-symmetry condition as was announced in the Introduction.

4. Spectrum of mesons

In this section we will study the quadratic fluctuations around the D7 brane configuration $z_4 = \mu$ for $\mu = 0$. We saw in the previous section that the holomorphic embedding $z_4 = \mu$ satisfies the κ-symmetry condition for arbitrary μ. For $\mu \neq 0$, however, the D7 brane probe action has a very complicated form and the meson spectrum calculation becomes an extremely difficult task. We will therefore consider only the $\mu = 0$ case, which in view of the equation (2.30) corresponds to massless quarks. In the absence of the quark mass parameter the meson masses should be normalized with respect to the QCD scale introduced by the conifold deformation parameter ϵ or alternatively with respect to the glueball mass (2.20).

The set of dynamical fields on the brane includes $8d$ gauge potentials and two $8d$ scalars corresponding to the fluctuations along the two transverse directions. We will consider the abelian case $N_\mathrm{f} = 1$ and will examine only the lowest KK modes, namely we will assume that the $8d$ fields depend only on the radial coordinate τ and on the D3 world-volume coordinates x_μ's. Fields with a non-trivial dependence on the angular coordinates associated with the 1-forms h_i's carry non-zero spins and charges related to the SU(2) isometry of the embedding and therefore have no counterparts in the dual QCD.

As we mentioned in the Introduction the masses of the fluctuation modes give the spectrum of the gauge theory mesons. There are two kinds of mesons. The gauge fields A_μ with non-vanishing components along the non-compact directions x_μ correspond to vector mesons in the gauge theory, while the gauge fields directed along the compact angular directions (A_1, A_2, A_3) and the scalar fields on the brane correspond to scalar mesons.

The D7 brane action consists of two parts:

$$S = S_{\mathrm{DBI}} + S_{\mathrm{WZ}}\,, \tag{4.1}$$

where

$$S_{\mathrm{DBI}} = -\mu_7 e^{-\phi_0} \int \mathrm{d}\sigma^8 \sqrt{\varphi^\star(g) + \mathcal{F}}\,, \qquad \text{and} \qquad S_{\mathrm{WZ}} = \mu_7 \int \sum_p C_{p+1} \wedge e^{\mathcal{F}}\,. \tag{4.2}$$

Here $\mu_7 = (2\pi)^{-7} l_s^8$, $e^{-\phi_0} = g_s^{-1}$, $\varphi^\star(g)$ is the pullback of the $10d$ metric (2.17) and $\mathcal{F} = \varphi^\star(B) + 2\pi l_s^2 F$ is the modified 2-form field strength as in the previous section. We saw in Section 2 that $C_6 = 0$ in the KS model. Moreover, C_0 also vanishes in this background and C_2 has no legs along the $4d$ space-time spanned by the coordinates x_μ's. Thus we conclude that the only contribution to the WZ part of the action is due to the RR 4-form C_4 (2.22).

It appears that up to the quadratic terms the D7 brane action takes the following form:

$$\begin{aligned}
S = -\mu_7 \int & \mathrm{d}^4x \mathrm{d}\tau h_1 h_2 h_3 \times \\
\times \Bigg[& \sqrt{-|g_{(0)}|} \Bigg(1 + \frac{1}{4}\left(1 + \frac{1}{\cosh\tau}\right) h^{1/2} A^2 g^{\mu\nu}_{(0)} \partial_\mu\gamma\partial_\nu\gamma + \frac{1}{4}\left(\frac{A}{B}\right)^2 (\partial_\tau\gamma)^2 - \\
& - \frac{1}{4}\left(1 - \frac{1}{4}\left(\frac{A}{B}\right)^2\left(1 - \frac{1}{\cosh\tau}\right)\right)\gamma^2 + \frac{1}{4} g^{IJ}_{(0)} g^{KL}_{(0)} \mathcal{F}_{IK}\mathcal{F}_{JL}\Bigg) - \\
& - \frac{1}{2!} h^{-1}\epsilon^{ijkl}\mathcal{F}_{ij}\mathcal{F}_{kl}\Bigg] .
\end{aligned} \tag{4.3}$$

Here μ, $\nu, \ldots$ refer to the $4d$ non-compact world-volume coordinates, $I, J, \ldots$ are the $8d$ indices, $i, j, \ldots = \tau, 1, 2, 3$ denote the radial and the internal space indices, the functions $A(\tau)$ and $B(\tau)$ are given in (2.9) and the $8d$ metric $g^{IJ}_{(0)}$ is:

$$g^{IJ}_{(0)} = \mathrm{diag}\left(-h^{\frac{1}{2}}, h^{\frac{1}{2}}, h^{\frac{1}{2}}, h^{\frac{1}{2}}, \frac{\epsilon^{-\frac{4}{3}}h^{-\frac{1}{2}}}{B^2}, \frac{\epsilon^{-\frac{4}{3}}h^{-\frac{1}{2}}}{2A^2\left(1 - \frac{1}{\cosh\tau}\right)}, \frac{\epsilon^{-\frac{4}{3}}h^{-\frac{1}{2}}}{2A^2\left(1 + \frac{1}{\cosh\tau}\right)}, \frac{\epsilon^{-\frac{4}{3}}h^{-\frac{1}{2}}}{4B^2}\right). \tag{4.4}$$

It implies that $\sqrt{-|g_{(0)}|} = 4A^2(\tau)B^2(\tau)\tanh\tau$. The components of the 2-form $\mathcal{F}$ appearing in the action (4.3) are:

$$\begin{aligned}
\mathcal{F}_{\tau 1} &= 2\pi l_s^2 F_{\tau 1} \qquad \mathcal{F}_{23} = 2\pi l_s^2 F_{23} \qquad \mathcal{F}_{\tau 3} = 2\pi l_s^2 F_{\tau 3} \qquad \mathcal{F}_{12} = 2\pi l_s^2 F_{12} \\
\mathcal{F}_{\tau 2} &= -\frac{1}{4} M l_s^2 (k(\tau) - f(\tau))(\cosh\tau + 1)\gamma' + 2\pi l_s^2 F_{\tau 2} \\
\mathcal{F}_{13} &= \frac{1}{4} M l_s^2 (k(\tau) - f(\tau))(\cosh\tau - 1)\gamma + 2\pi l_s^2 F_{13} \\
\mathcal{F}_{\mu 1} &= 2\pi l_s^2 F_{\mu 1} \qquad \mathcal{F}_{\mu 3} = 2\pi l_s^2 F_{\mu 3} \\
\mathcal{F}_{\mu 2} &= \frac{1}{4} M l_s^2 (k(\tau) - f(\tau))(\cosh\tau + 1)\partial_\mu\gamma + 2\pi l_s^2 F_{\mu 2} .
\end{aligned} \tag{4.5}$$

In writing these components we used (2.23). We see that the action (4.3) has no terms linear in the fields. Hence the supersymmetric configuration $z_4 = 0$ solves the probe equations of motion. It is interesting to notice that the field $\delta(x_\mu, \tau)$ does not appear in the action. A simple check reveals that the term proportional to δ^2 will be produced in the $\mu \neq 0$ case.

From the expressions for $\mathcal{F}_{13}$ and $\mathcal{F}_{\tau 2}$ it is evident that there is a mixing between the scalar field γ and the A_2 component of the gauge field.

In the rest of this section we will use the action (4.3) to calculate the masses of $4d$ vector and scalar mesons. In what follows we will impose the gauge $A_\tau = 0$.

4.1 Vector mesons

For the vector field polarized along the non-compact world-volume coordinates we will adopt the ansatz $A_\mu(x, \tau) = v_\mu e^{ik\cdot x} a(\tau)$, where $-k^2 = M_{\mathrm{m}}^2$ denotes the $4d$ mass. The

JHEP03(2005)014

equation of motion extracted from (4.3) is:

$$\partial_c\left(\sqrt{g}g^{cd}g^{ab}F_{db}\right)=0\,, \tag{4.6}$$

where the indices $a,b,\ldots$ run over the radial τ and the $4d$ coordinates x_μ and g_{ab} corresponds to an appropriate metric component in (4.4). It implies that $v\cdot k=0$ and the function $a(\tau)$ satisfies:

$$\left(A^2(\tau)\tanh(\tau)a'(\tau)\right)'+\lambda_n^2 I(\tau)A^2(\tau)B^2(\tau)\tanh(\tau)a(\tau)=0\,. \tag{4.7}$$

Here $\lambda_n=M_{\rm m}/m_{\rm gb}$ with $m_{\rm gb}$ being the scale mass of the gauge theory glueballs (2.20) and $I(\tau)$ is defined in (2.19). The normalizable solution of the above differential equation at $\tau\to\infty$ is $a(\tau)\sim e^{-2\tau/3}$. On the other hand, the converging solution near $\tau=0$ is $a(\tau)\sim$ const. Matching numerically the regular solutions in the UV ($\tau\to\infty$) and in IR ($\tau\to 0$) results in a set of discrete values of the parameter λ_n. This computation leads to the following result for the lowest masses of vector mesons in the dual gauge theory:

$$\lambda_n=\frac{M_{\rm m}}{m_{\rm gb}}=4.32,\ 5.81,\ 7.32,\ 8.85,\ 10.39,\ldots\,. \tag{4.8}$$

4.2 Scalar mesons

We next consider the gauge field A_1 and A_3. Substituting $A_1(x,\tau)=a_1(\tau)e^{ik\cdot x}$ and $A_3(x,\tau)=a_3(\tau)e^{ik\cdot x}$ into the action we end up the following equations for $a_{1,3}(\tau)$:

$$\left(\frac{\coth\left(\frac{\tau}{2}\right)}{I(\tau)}a_1'(\tau)\right)'=\left[\frac{1}{2}\left(\frac{1}{I(\tau)}\right)'+\frac{1}{4}\frac{\tanh\left(\frac{\tau}{2}\right)}{I(\tau)}-\lambda_n^2B^2(\tau)\coth\left(\frac{\tau}{2}\right)\right]a_1(\tau) \tag{4.9}$$

and

$$\left(\frac{A^2(\tau)}{I(\tau)B^2(\tau)}a_3'(\tau)\right)'=\left[\left(\frac{1}{I(\tau)}\right)'+\frac{B^2(\tau)}{I(\tau)A^2(\tau)}\coth(\tau)-\lambda_n^2A^2(\tau)\tanh(\tau)\right]a_3(\tau)\,. \tag{4.10}$$

Now the regular solutions at $\tau\to\infty$ are $a_1(\tau)\sim e^{-11/6\tau}$ and $a_3(\tau)\sim e^{-2\tau}$, while at $\tau\to 0$ the converging solutions are $a_1(\tau)\sim\tau^2$ and $a_3(\tau)\sim\tau$. This time the "shooting" technique yields the following spectra:

$$\lambda_n=\frac{M_{\rm m}}{m_{\rm gb}}=3.38,\ 4.92,\ 6.38,\ 7.87,\ 9.37,\ldots \tag{4.11}$$

and

$$\lambda_n=\frac{M_{\rm m}}{m_{\rm gb}}=4.18,\ 5.67,\ 7.17,\ 8.71,\ 10.28,\ldots \tag{4.12}$$

for the mesons related to the field A_1 and A_3 respectively.

JHEP03(2005)014

Finally let us consider the fields γ and A_2. Following the same steps as in the previous discussion we we will assume that $\gamma = \gamma(\tau)e^{ik\cdot x}$ and $A_2 = a_2(\tau)e^{ik\cdot x}$. We arrive to a set of two 2nd order differential equations for $\gamma(\tau)$ and $a_2(\tau)$:

$$\begin{aligned}
&\left(I^{-1}\tanh\frac{\tau}{2}\left(-(k-f)(\cosh\tau+1)\gamma'+\tilde{a}_2'\right)+\frac{1}{2}I^{-1}\left((k-f)(\cosh\tau-1)\gamma-\tilde{a}_2\right)\right)'=\\
&\quad=\frac{1}{4}I^{-1}\coth\frac{\tau}{2}\left(-(k-f)(\cosh\tau-1)\gamma+\tilde{a}_2\right)+\frac{1}{2}I^{-1}\left((k-f)(\cosh\tau-1)\gamma'-\tilde{a}_2'\right)-\\
&\quad\quad-\lambda_n^2\left(\frac{B}{A}\right)^2\tanh\frac{\tau}{2}\left((k-f)(\cosh\tau+1)\gamma+\tilde{a}_2\right)
\end{aligned}\tag{4.13}$$

and

$$\begin{aligned}
&\Bigg(I^{-1}(k-f)\Big((\cosh\tau+1)\tanh\frac{\tau}{2}\left((k-f)(\cosh\tau+1)\gamma'-\tilde{a}_2'\right)+\\
&\quad+\frac{1}{2}(\cosh\tau-1)\left(-(k-f)(\cosh\tau-1)\gamma+\tilde{a}_2\right)\Big)+2^{\frac{14}{3}}A^4\tanh\tau\,\gamma'\Bigg)'=\\
&=\frac{1}{2}I^{-1}(k-f)(\cosh\tau-1)\Bigg(\frac{1}{2}\coth\frac{\tau}{2}((k-f)(\cosh\tau-1)\gamma-\tilde{a}_2)-\\
&\qquad\qquad-\left((k-f)(\cosh\tau-1)\gamma'-\tilde{a}_2'\right)\Bigg)-\\
&\quad-\lambda_n^2B^2(k-f)(\cosh\tau+1)\tanh\frac{\tau}{2}((k-f)(\cosh\tau+1)\gamma+\tilde{a}_2)-\\
&\quad-2^{14/3}A^2B^2\tanh\tau\left(-1+\frac{1}{4}\left(\frac{A}{B}\right)^2\left(1-\frac{1}{\cosh\tau}\right)-\lambda_n^2IA^2\left(1-\frac{1}{\cosh\tau}\right)\right)\gamma\,.
\end{aligned}\tag{4.14}$$

Here $\tilde{a}_2 \equiv \frac{8\pi}{M}a_2$. The regular solution at $\tau\to\infty$ behaves as $(\tau\gamma(\tau)-\tilde{a}_2)\sim e^{-11/6\tau}$ and at $\tau\to 0$ we have $\gamma(\tau), a_2(\tau)\sim$ const. Solving numerically this set we get:

$$\lambda_n = \frac{M_{\rm m}}{m_{\rm gb}} = 4.01,\ 5.52,\ldots\,. \tag{4.15}$$

5. Discussion

In this paper we have investigated holomorphically embedded D7 branes in the KS background in order to get a supergravity description of $\mathcal{N}=1$ QCD with flavors. We considered the $N_{\rm f}=1$ case and ignored the back-reaction of the D7 brane probe on the supergravity background. Studying the κ-symmetry condition derived in [35] we argued that our brane configuration $z_4=\mu$ preserves the background supersymmetry. We also gave a simple criterion to verify whether holomorphic embeddings suggested by other authors are supersymmetric. The embedding $z_4=\mu$ discussed in this paper preserves the SU(2) subgroup of the deformed conifold isometries. As we have shown this fact significantly simplifies the choice of the coordinates along the D7 brane and transversal to it. Using this coordinate parameterization we calculated the spectrum of the vector and scalar mesons for the special $\mu=0$ case. To this end we studied the equations of motion extracted from the quadratic

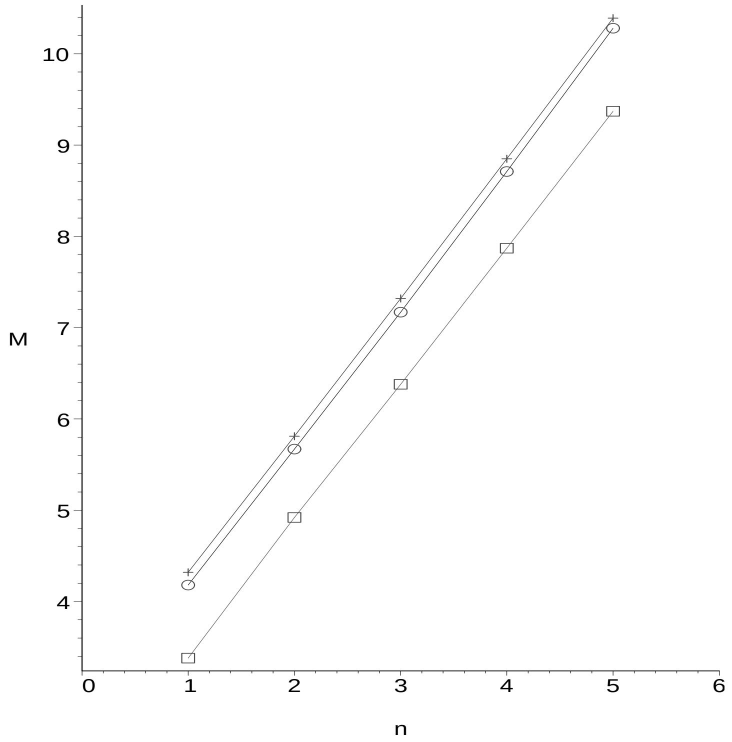

Figure 2: Graphic representation of the meson spectrum for $m_{\rm gb} = 1$ and $n = 1, \ldots, 5$. The boxes ($\square$) and the circles ($\circ$) correspond to the scalar meson masses (4.11) and (4.12) respectively, while the crosses ($+$) are related to the vector mesons (4.8)

fluctuations around the probe. The meson masses were computed this way by numerical matching of the regular solutions in the UV and in the IR. The graphical representation of the spectrum is given on Fig. 2. We found that the spectrum satisfies the following important properties:

- It exhibits a mass gap.
- The masses are of the order of the glueball mass $m_{\rm gb}$.
- The vector mesons are heavier than the corresponding scalar mesons. Remarkably, this property is not shared by the meson spectrum found in [24].
- At least for the lowest levels $n = 1, \ldots, 5$ we have obtained here, the masses $M_{\rm m}$ of the vector and the scalar mesons grow linearly as a function of n. In all the cases (4.8), (4.11) and (4.12) we have found that:

$$M_{\rm m}(n) = \Delta m \cdot n + a_0 \,, \tag{5.1}$$

where $\Delta m \approx 1.5 m_{\rm gb}$ and $a_0 = 2.8 m_{\rm gb}$, $1.9 m_{\rm gb}$, $2.7 m_{\rm gb}$ for the cases (4.8), (4.11) and (4.12) respectively.

JHEP03(2005)014

Unfortunately, we don't have any physical explanation of this linear behavior. Very remarkably, the same phenomena was also observed in [33], where the meson spectrum was calculated from the D5 probe brane in the MN background (see [33, (7.24)] for $l = 0$).

- For $\mu = 0$ there is no equation associated with the scalar field δ, since this field does not contribute to the quadratic fluctuations of the probe. This is very similar to the results of [24]. This feature certainly has to do with the fact that for $\mu = 0$ there is a residual U(1) isometry along the D7 probe brane as we have mentioned in section 2.

There are plenty of open problems to be explored. In this paper we computed the meson mass spectrum only for the $\mu = 0$ case. It will be very interesting to find the spectrum for $\mu > 0$. This spectrum will be characterized by two mass scales: μ and the glueball mass related to the conifold parameter ϵ. Although we saw that the embedding $z_4 = \mu$ is supersymmetric for any μ, it might still be possible that this configuration does not solve the probe equations of motion. It is also important to perform the spectrum computation for other holomorphic configurations like (1.2) and (1.4) and to compare it to the results of this paper.

In [56] an analytic first order deformation of the KS background was found. From the dual gauge theory point of view this deformation describes supersymmetry soft breaking gaugino mass terms. It will be interesting to consider a D7 brane probing the non-supersymmetric background and to understand how the deformation modifies the meson spectrum we have obtained here.

Finally, it is very desirable to find a modification of the KS solution which incorporates fully localized D7 branes. This is needed for studying gauge theories where the approximation $N_{\mathrm{f}} \ll N_{\mathrm{c}}$ is not justified. First steps in this direction have been made in [26] and [20].

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A. The explicit expressions for the 1-forms h_i and $\tilde{h}_i$

Here we give the expressions for the form h_i, $\tilde{h}_i$ $(i = 1, 2, 3)$ in terms of the five angular coordinates θ_1, ϕ_1, θ_2, ϕ_2 and ψ:

$$\begin{aligned}
h_1 &= -\cos\frac{\psi}{2}\sin\theta_1 \mathrm{d}\phi_1 + \sin\frac{\psi}{2}\mathrm{d}\theta_1 \\
h_2 &= -\sin\frac{\psi}{2}\sin\theta_1 \mathrm{d}\phi_1 - \cos\frac{\psi}{2}\mathrm{d}\theta_1 \\
\tilde{h}_1 &= -\cos\frac{\psi}{2}\sin\theta_2 \mathrm{d}\phi_2 + \sin\frac{\psi}{2}\mathrm{d}\theta_2
\end{aligned}$$

$$\begin{aligned}\tilde{h}_2 &= -\sin\frac{\psi}{2}\sin\theta_2 \mathrm{d}\phi_2 - \cos\frac{\psi}{2}\mathrm{d}\theta_2 \\ h_3 + \tilde{h}_3 &= \mathrm{d}\psi + \cos\theta_1 \mathrm{d}\phi_1 + \cos\theta_2 \mathrm{d}\phi_2 .\end{aligned} \tag{A.1}$$

B. The connection between the forms $g_{i=1,\dots,5}$ and the forms $h_{i=1,2,3}$, $\tilde{h}_{i=1,2,3}$

$$\begin{aligned}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} &= \begin{pmatrix} \cos\frac{\psi}{2} & \sin\frac{\psi}{2} \\ \sin\frac{\psi}{2} & -\cos\frac{\psi}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}(g_1+g_3) \\ \frac{1}{\sqrt{2}}(g_2+g_4) \end{pmatrix} \\ \begin{pmatrix} \tilde{h}_1 \\ \tilde{h}_2 \end{pmatrix} &= \begin{pmatrix} -\cos\frac{\psi}{2} & -\sin\frac{\psi}{2} \\ \sin\frac{\psi}{2} & -\cos\frac{\psi}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}(g_3-g_1) \\ \frac{1}{\sqrt{2}}(g_4-g_2) \end{pmatrix} \\ \text{and} \qquad h_3 + \tilde{h}_3 &= g_5 \,. \end{aligned} \tag{B.1}$$

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JHEP03(2005)014

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